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Estimation of Probabilities of Large Declines in Bitcoin Prices and Their Application to Lending Transactions

Abstract

USD and other fiat loans may be collateralized with cryptocurrency to manage credit risk. A version of this transaction also adds additional margin that is escrowed by a third party. For a loss to occur, two things need to happen: the counterparty has to renege on its legal obligation AND the collateral has to fall in value in excess of the escrowed margin. The purpose of this paper is to assess the likelihood of BTC falling in value with a magnitude high enough to make a loss possible. It is beyond the scope of this paper to determine the likelihood of default -- although indications are that crypto businesses do honor their commitments. Nevertheless, the goal here is to assess maximum expected potential loss or "the worst-case scenario" and how that translates into a threshold yield to earn.

We examine the potential default and loss rates from the perspective of a USD investor in a loan collateralized by BTC. We use three methodologies: historical observation, parametric observation based on historical data, and parametric observation using options data. We find that a reasonable model for the behavior of BTC is a Student's t-distribution with 4-6 degrees of freedom leading to an indicated worst-case (assuming 100% counterparty renege) loss rate and, therefore, threshold investment rate of return, of no more than 1-5%. Further, the availability of options markets allows the potential for hedging that risk.

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Introduction

Bitcoin collateralized lending transactions are a rapidly expanding business. A USD investor interested in such a transaction faces counterparty credit risk (as does the USD borrower that has collateralized the loan using BTC). While it is difficult to judge the credit risk of participants in the crypto lending market due to lack of information, one can apply quantitative techniques to the collateral to determine what can be called "backstop" risk in the event of counterparty default. The salient question for a USD investor is the likelihood of a catastrophic drop in BTC occurring before the next mark-to-market or margin call.

To that end, we propose three methodologies to estimate the probability of a large decline in the price of bitcoin daily utilizing both historical data and liquidly traded Deribit options. The potential risk and reward of a bitcoin lending transaction can then be estimated considering different levels of initial collateralization. The results, though sensitive to initial assumptions and parameterizations, indicate that the risk/reward of a bitcoin lending transaction may be quantified with the ability to estimate the probability of a large decline in bitcoin over a short time period, say, one day, matching the period over which margin calls are made.

Bitcoin lending transaction

We first define an occurrence that would characterize an event of loss for a bitcoin lending transaction between 2 counterparties which involves inherent credit risk. For simplicity, we define an event of potential loss (EPL) as a daily decline of the price of bitcoin exceeding a certain percentage that will result in loss for a USD investor should the borrower of USD default on the transaction. For a typical transaction, the borrower receives USD from the lender. The lender of USD receives the equivalent bitcoin amount from the lender based on the price on the day of transaction. Both lender and borrower also post a certain percentage of initial collateralization to a third-party (such as Digital Gamma). As such, the main risk faced by the lender in the transaction is the combined risk that bitcoin prices decline significantly and the borrower defaults, potentially resulting in a loss if the decline exceeds that of the collateral that has been posted. The resulting loss on the trade then depends on the amount of the initial collateral that is in escrow from the previous day of rebalance. Hence, our focus is on quantifying the probability of large declines in bitcoin prices to quantify the risk to a bitcoin lender should their counterparty violate the transaction and default. A typical excess margin (overcollateralization) is 25% and, therefore, 25% is the significant decline that this research focuses on.

We can estimate the probability of a daily decline exceeding a certain percentage in three different ways.

1. Based on historical data. For example, we have daily data available from Kaiko for 4 different exchanges (Gemini, Bitstamp, Coinbase and Kraken) from Oct 8, 2015 to May 13, 2020. We formed a composite index based on the average of the daily prices from all 4 exchanges. We only have 1 day (out of 1679 total days) where we have a daily decline that exceeds 25% and 30% (March 12 2020 daily decline of 39%), and 5 days where we have a daily decline that exceeds 15% or more. As such, based on these data, we can then infer the probability of a daily decline that exceeds 15% or more to be 5/1679 = 3.0×10^{-3}, and the probability of a daily decline that exceeds 20% or more and 25% or more to be 1/1679 = 6.0×10^{-4}. Of course, because the available data is limited, this approach may not sufficiently represent the true probability distribution function of the daily decline percentage. This is the reason why we further introduce parametric approaches based on fitting standard probability distribution functions to the means and standard deviations obtained from historical data, from which one can then use to infer the daily probabilities of declines.
2. Based on parametric approaches as alluded to above. For example, we can fit normal/Student's t-distributions to the historical data, from which we can infer the probabilities of daily decline. We will explain more in a later section.
3. Based on liquid option prices that trade. For example, the deltas of the puts traded on the Deribit platform provide an estimation of the probability that the price of bitcoin will decline below the strike on expiration date. We can then use this information to deduce the probabilities of daily declines. We will explain more in a later section.

Parametric Approach to estimate the probability of the daily decline percentage

The parametric approach to estimate the daily probability of EPL (defined as a daily percentage decline of bitcoin exceeding a certain percentage) is highly dependent on:

1. The type of probability distribution function that is used to model the daily returns
2. Once the probability distribution function is selected, the parameters for the probability distribution functions need to be estimated from the historical daily data of bitcoin prices, and

that will be highly dependent on the length of data that is chosen to calibrate the required parameters.

We chose to use a Student's t-distribution of various degrees of freedom to model bitcoin prices. The composite bitcoin prices from Kaiko are used as our data source, and the data starts from Oct 8, 2015 to May 13, 2020. For each day, we used the data from the prior 30 days on a rolling basis to estimate the mean and standard deviation of the daily log-returns as parameters to be used for both the normal distribution and the Student's t-distributions.

Using a normal distribution to model the daily returns, the estimated daily EPL, if we define the EPL event as a daily decline exceeding 25%, is approximately 10^{-10} (1 day in 27 million years). This is because the normal distribution has very thin tails which makes it unsuitable for modeling the probability of extreme events. A Student's t-distribution with a low degree of freedom will have a much fatter tail and we perform the same calculation of daily EPL probability for different degrees of freedom for the Student's t-distributions and different exceedance percentages for the daily return. Here are the results.

	Daily prob of default for different dof in t-distribution and tolerance in bitcoin scenario					
	Tolerance →					
	1.20E-05	-30%	-25%	-20%	-15%	-10%
t-dof	2	9.0E-03	1.3E-02	1.9E-02	3.1E-02	6.0E-02
↓	4	9.1E-04	1.7E-03	3.8E-03	9.5E-03	2.9E-02
	6	1.6E-04	4.2E-04	1.2E-03	4.4E-03	2.0E-02
	8	4.0E-05	1.3E-04	5.3E-04	2.6E-03	1.5E-02
	10	1.2E-05	5.3E-05	2.7E-04	1.7E-03	1.3E-02
	15	1.2E-06	8.9E-06	8.1E-05	8.6E-04	9.5E-03
	20	2.1E-07	2.5E-06	3.5E-05	5.5E-04	8.1E-03
	30	1.7E-08	4.3E-07	1.2E-05	3.2E-04	6.7E-03

It may be difficult to conceptualize the very small probabilities from our analysis. An alternative way to think about such small numbers is to quantify how many days on average needs to elapse before we expect to experience a decline of that magnitude. This is typically referred to as the return period or recurrence period in statistics. The return periods can be simply calculated by taking the reciprocal of the daily decline probabilities above. Here are the same results presented using return periods (in days).

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	Return periods (in days) under different assumptions					
	Tolerance					
		-30%	-25%	-20%	-15%	-10%
t-dof	2	111	79	53	32	17
	4	1099	572	265	105	34
	6	6183	2407	808	225	51
	8	25130	7482	1877	387	66
	10	81949	19019	3671	582	79
	15	837548	112058	12399	1167	105
	20	4798039	402509	28473	1817	124
	30	58488330	2321385	83587	3103	149

The above methodology is a parametric approach where we used the historical data to fit a specific probability distribution function from which to estimate the tail probabilities. Of course, the time window used to estimate the parameters can also be varied to examine the impact on the probability estimations. To match the probability of daily decline that exceeds 25% estimated using historical data (i.e., 6×10^{-4} from an earlier section), the degrees-of-freedom of the Student's t-distribution is approximately between 4 and 6.

Market-based Approach to estimate the probability of the daily decline percentage

Alternatively, one can also adopt a market-based approach to estimate the probability of the daily decline exceeding a certain percentage from option deltas as well.

For example, on May 13, 2020, the Deribit platform has Europeans options with expirations May 22 2020, June 26 2020 and Sep 25 2020. If the price of bitcoin were to decline by 25% that day, it will fall to about $7000. The puts struck at $7000 have deltas of 0.07, 0.17 and 0.215 for option expirations May 22 2020, June 26 2020 and Sep 25 2020, and they are an indication of the probability that the puts will expire in the money on those specific expiration dates.

If we were to model the event of a daily decline of greater than 25% as a Poisson event, this would equate to intensity of 2.8, 1.4 and 0.6 events per year, from which we can compute the probability of a daily decline exceeding 25% to be 7.7×10^{-3}, 3.9×10^{-3} and 1.6×10^{-3} respectively based on the option expirations May 22 2020, June 26 2020 and Sep 25 2020. Similarly, for other daily decline percentages, we can look at other strikes that match the desired daily decline percentages (or interpolate to get the required strikes), and the same procedure above can then be used to estimate the probabilities.

Based on this market-based options approach, to match the probability of daily decline that exceeds 25% estimated using Deribit put options, the degrees-of-freedom of the Student's t-distribution is approximately between 2 and 4.

Summary of results of daily EPL probabilities

Here is a summary of the estimated probabilities from the three different approaches:

		Daily decline probabilites under different assumptions						
Daily % drop		Historical from 9/17/2014	T, 2 dof	T, 4 dof	T, 6 dof	T, 8 dof	Jun 26 Options	Sep 25 Options
	-30%	6.0E-04	9.0E-03	9.1E-04	1.6E-04	4.0E-05	3.1E-03	1.3E-03
	-25%	6.0E-04	1.3E-02	1.7E-03	4.2E-04	1.3E-04	3.9E-03	1.6E-03
	-20%	6.0E-04	1.9E-02	3.8E-03	1.2E-03	5.3E-04	4.8E-03	1.8E-03
	-15%	3.0E-03	3.1E-02	9.5E-03	4.4E-03	2.6E-03	5.7E-03	2.1E-03

One can see that perhaps because of the limited availability of historical data, the parametric or market-based approach to estimate the probabilities of daily decline percentages that exceed 25% or more is considerably higher as compared to what we observed using historical data. One can also see that the estimated probabilities from the market-based approach using deltas from liquidly-traded Deribit puts are considerably higher than the parametric approach, unless we consider a degree of freedom to be approximately 2 or 3 for the Student's t-distribution. A conservative approach that takes into account the increased likelihood of large declines in the price of bitcoin may be appropriate, especially given the recent increased proliferation of lending platforms that may have exacerbated, in a "negatively-convex" manner, the significant volatility in bitcoin prices in times of market stress (such as what we observed on March 12 2020). Also, as a gauge for comparison purposes, the daily default probabilities of fixed-income investment-grade and high-yield corporate bonds are 4.3×10^{-5} and 2.4×10^{-4} respectively.

Similarly, here is a summary of the return periods using the 3 approaches as above.

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Daily % drop		Return periods (in days) under different assumptions						
		Historical from 9/17/2014	T, 2 dof	T, 4 dof	T, 6 dof	T, 8 dof	Jun 26 Options	Sep 25 Options
	-30%	1,667	111	1,099	6,183	25,000	323	769
	-25%	1,667	77	588	2,407	7,692	256	625
	-20%	1,667	53	263	808	1,887	208	556
	-15%	336	32	105	225	385	175	476

Given the above estimated probabilities, one can also estimate the annualized yields required from the transaction to compensate the lender for different levels of losses upon default. For example, if the daily decline percentage is 30% and we put up 25% initial margin, then in the worst-case scenario that every counterparty has defaulted, then the loss is 5%. The estimated annualized yield should be approximately equal to the product of the probability that we have a daily decline percentage that exceeds 25% multiplied by the loss percentage. Here is a summary of the annualized fair annual threshold yields for different levels of loss percentages using the probabilities we obtained from the three different approaches.

Annual Threshold Yields for daily percentage drop exceeding 25% and different levels of losses

Potential Loss	Historical from 9/17/2014	T, 2 dof	T, 4 dof	T, 6 dof	T, 8 dof	Jun 26 Options	Sep 25 Options
2%	0.4%	2.0%	0.9%	0.3%	0.1%	1.5%	0.9%
4%	0.8%	4.0%	1.8%	0.6%	0.2%	3.0%	1.8%
6%	1.2%	5.9%	2.8%	0.8%	0.3%	4.6%	2.7%
8%	1.6%	7.9%	3.7%	1.1%	0.4%	6.1%	3.5%
10%	2.0%	9.9%	4.6%	1.4%	0.5%	7.6%	4.4%
12%	2.4%	11.9%	5.5%	1.7%	0.6%	9.1%	5.3%
14%	2.8%	13.9%	6.5%	2.0%	0.6%	10.6%	6.2%
16%	3.1%	15.9%	7.4%	2.3%	0.7%	12.1%	7.1%
18%	3.5%	17.8%	8.3%	2.5%	0.8%	13.7%	8.0%
20%	3.9%	19.8%	9.2%	2.8%	0.9%	15.2%	8.8%

Of course, for the above analysis, we are assuming that the borrower will not be able to make up the losses once the percentage decline in bitcoin exceeds that of the collateral that he has already posted at the third party. For example, if the daily decline percentage is 30% and he has already posted 25% initial collateral, then, we assume that the entire excess amount is lost, i.e., the loss is 5%. This is a conservative assumption. If, for example, he could post another 2% to the third party, then the actual loss is only 3% and we must readjust our analysis as such.

Also, our analysis assumes that the collateral is posted in USD. If collateral is posted instead in BTC, then the value of the BTC collateral will also reduce as the value of bitcoin declines. Consequently, the potential loss will also increase. We reproduce the same table shown earlier with new columns that indicate what the implied levels of collateral will be in both USD and BTC percentages for levels of potential losses ranging from 2% to 20%.

Annual Threshold Yields for 25% decline of Bitcoin, different levels of losses and the corresponding implied collateral %									
Potential Loss	Implied USD collateral %	Implied BTC collateral %	Historical from 9/17/2014	T, 2 dof	T, 4 dof	T, 6 dof	T, 8 dof	Jun 26 Options	Sep 25 Options
2%	23%	30.7%	0.4%	2.0%	0.9%	0.3%	0.1%	1.5%	0.9%
4%	21%	28.0%	0.8%	4.0%	1.8%	0.6%	0.2%	3.0%	1.8%
6%	19%	25.3%	1.2%	5.9%	2.8%	0.9%	0.3%	4.6%	2.7%
8%	17%	22.7%	1.6%	7.9%	3.7%	1.1%	0.4%	6.1%	3.5%
10%	15%	20.0%	2.0%	9.9%	4.6%	1.4%	0.5%	7.6%	4.4%
12%	13%	17.3%	2.4%	11.9%	5.5%	1.7%	0.6%	9.1%	5.3%
14%	11%	14.7%	2.8%	13.9%	6.5%	2.0%	0.6%	10.6%	6.2%
16%	9%	12.0%	3.1%	15.9%	7.4%	2.3%	0.7%	12.1%	7.1%
18%	7%	9.3%	3.5%	17.8%	8.3%	2.6%	0.8%	13.7%	8.0%
20%	5%	6.7%	3.9%	19.8%	9.2%	2.8%	0.9%	15.2%	8.8%

Similarly, if BTC collateral is posted instead of USD collateral, the fair annual threshold yields should consequently be higher as well. The results in the following table is a summary of the values of these yields for a daily decline in bitcoin from 15% to 50% using a Student's t-distributions with 6 degrees of freedom and levels of USD and BTC collateral from 15% to 30%.

				Fair Annual Threshold Yields							
		T, 6 dof		% USD Collateral				% BTC Collateral			
Daily % drop		Daily probability of decrease	Return Period in days	15%	20%	25%	30%	15%	20%	25%	30%
	50%	0.0010%	102484	0.12%	0.11%	0.09%	0.07%	0.15%	0.14%	0.13%	0.12%
	40%	0.0034%	29352	0.31%	0.25%	0.19%	0.12%	0.38%	0.35%	0.31%	0.27%
	30%	0.0162%	6183	0.86%	0.57%	0.29%	0.00%	1.12%	0.92%	0.72%	0.52%
	25%	0.0415%	2407	1.41%	0.70%	0.00%	0.00%	1.93%	1.41%	0.88%	0.35%
	20%	0.1238%	808	1.82%	0.00%	0.00%	0.00%	2.91%	1.45%	0.00%	0.00%
	15%	0.4440%	225	0.00%	0.00%	0.00%	0.00%	1.80%	0.00%	0.00%	0.00%

Conclusion

We proposed three methodologies to estimate the probability of a large daily decline in the price of bitcoin using a combination of historical data and liquidly traded Deribit option prices. The results, together with assumptions of losses upon such large declines, allow us to quantify the annualized yield that would be appropriate from a bitcoin lending transaction to compensate for losses that would be incurred should the borrower default due to a potential large decline in the price of bitcoin.